Morphology of Kinetic Asymptotic Grids

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Abstract. This paper investigates the kinetic behaviour of asymptotic lamella grids with variable surface topology. The research is situated in the field of semi-compliant grid mechanisms. Novel geometric and structural simulations allow to control and predict the curvature and bending of lamellas, that are positioned either flat (geodesic) or upright (asymptotic) within a curved grid. We build upon existing research of asymptotic gridshells and present new findings on their morphology. We present a digital and physical method to design kinetic asymptotic grids. The physical experiments inform the design, actuation strategy and kinetic boundaries, and become a benchmark for digital results. The kinetic behaviour of each sample is analysed through five stages. The digital models are used to calculate the total curvature at every stage, map the energy stored in the elastic grids and predict equilibrium states. This comparative modelling method is applied to seven asymptotic grids to investigate transformations and the impact of singularities, supports and constraints on the kinetic behaviour. Open grids without singularities are most flexible and require additional, external and internal constraints. The cylindrical typology acts as a constraint and creates symmetric kinetic transformations. Networks with one, two and four singularities cause increasing rigidity and limit the kinetic transformability. Finally, two prototypical architectural applications are introduced, an adaptive shading facade and a kinetic umbrella structure, that show the possible scale and actuation of kinetic designs.

Keywords: Asymptotic networks · Semi-compliant mechanism · Kinetic behaviour · Comparative modelling

1 Introduction

Transformable structures are 4-dimensional and offer to design through time, beyond the static, and adapt to environmental conditions, structural influences or user's needs. We can distinguish between conventional rigid-body mechanisms utilizing hinges or telescopes, and compliant mechanisms (Howell 2002), which utilize the elastic properties of the material to perform a smooth change in curvature and store strain energy. This research is focused on a hybrid (semi-compliant) typology of kinetic grid mechanisms (Schikore et al. 2020), coupling elastic slats with scissor joints in a doubly-curved quadrilateral grid. The paper combines insights from architectural geometry and structural engineering (Fig. 1).

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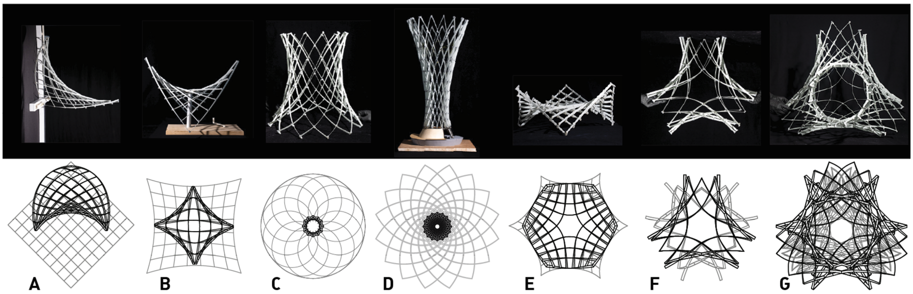


Fig. 1. Seven experimental asymptotic grids with variable topology. A) Regular grid (used for adaptive shading façade), B) Hypar grid, C) Regular rotational grid, D) Rotational grid with support constraints (Kinetic Umbrella), E) Schwarz D grid with one singularity, F) Tricylinder with two singularities, G) Quadcylinder with four singularities.

There is beautiful accordance of the mechanical behaviour of elastic slats in a grid, with the geometric properties of curves on a surface. Some of which have been described as early as 1897 by the mathematician Sebastian Finsterwalder (Finsterwalder 1897). This approach has gained momentum through the field of Architectural Geometry (Pottmann et al. 2007), which – among many other insights - brought forward methods to design doubly curved grids from developable strips (Tang et al. 2016). The approach is based on the theory of the *curvature of curves on a surface*. As long as the curves are attached continuously to the surface, and are measured in respect to the surface normal, their three local normal curvature k_n, geodesic curvature k_g and geodesic torsion τ_g can be calculated and correspond to the local deformation of an elastic profile (bending around **y**, bending around **z**, and torsion around **x**) within a gridshell (Schling and Barthel 2020). This knowledge has been applied to architectural construction by designing lamella gridshells along the asymptotic curves of anticlastic surface (Schling et al. 2017). Such asymptotes exhibit zero normal curvature ($k_n = 0$) and can thus be assembled from exclusively straight slats. The slats are positioned upright in the grid and adhere to the design geometry solely by twisting and bending around their weak axis. During the construction of the first steel prototype (Fig. 2), it was discovered that the asymptotic lamella grid was sufficiently constrained to deform predictably, following a semi-compliant mechanism (Schling et al. 2018). The complete lamella grid was assembled flat and transformed into the designated design shape purely by push of the hand.

Fig. 2. Kinetic behaviour of the first asymptotic steel grid. The lamellas are assembled flat and transform into the design shape simply by a push of the hand. They are constrained by the strong axis of lamellas and the scissor joints.

The phenomenon of controlled deployable gridshells has enjoyed some attention in the computational design community (Soriano et al. 2019; Isvoranu et al. 2019; Haskell et al. 2021). Both G-shells and X-shells focused on geodesic lamellas, tangential to the design grid. (Schikore et al. 2020) developed a typology, linking the kinetic behaviour to the three curvatures (k_n, k_g and τ_g) and corresponding profile axis (**x**, **y** and **z**) and used iso-geometric analysis (IGA) (Cottrell et al. 2009) to simulate the kinetic grids within the smooth NURBS environment. This publication also evaluated the necessary constraints to control a regular and a cylindrical asymptotic grid and introduced the curvature-square graph to allow designers to track the kinetic energy stored within throughout its transformation and predict its natural equilibrium state of minimum energy.

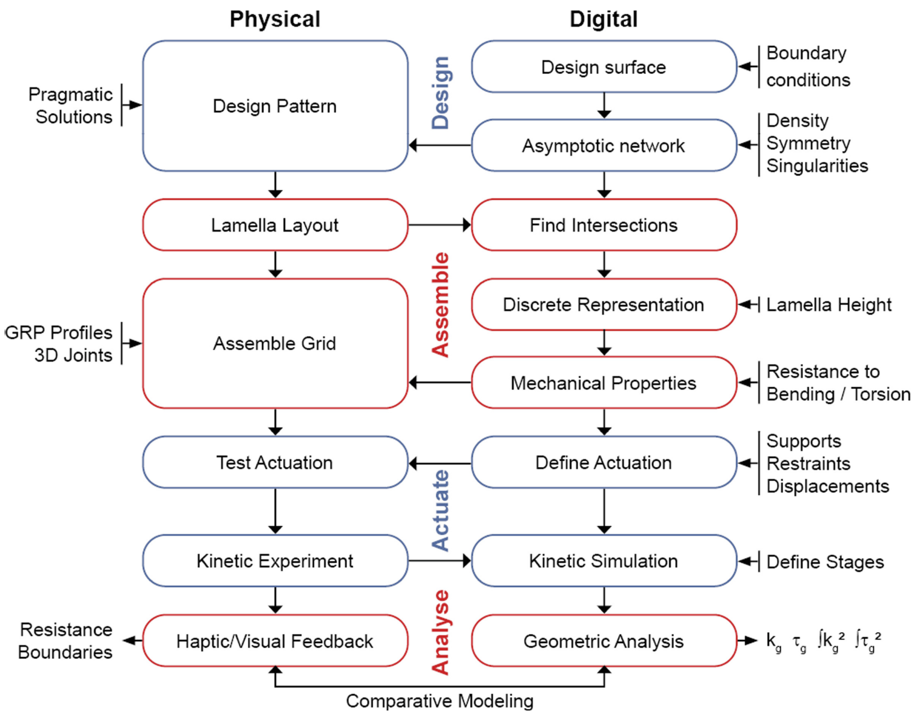


Fig. 3. Comparative Modelling. A hybrid approach is used, in which physical experiments and digital simulations are conducted in parallel to inform each other.

Contributions. The goal of this research is to extend the design language of kinetic asymptotic grids, by systematically investigating the impact of grid morphology on kinetic behaviour. The research is using a hybrid method of digital and physical experiments to design and evaluate 7 networks. Its feasibility for architecture is demonstrated through two design implementations. The work was first presented as an interactive digital and physical exhibition (https://eikeschling.com/2021/09/02/pmq-exhibition-kinetic-grid-structures/) in Hong Kong in September 2021 (Schling and Schikore 2021).

The paper presents a method to build accurate physical models of elastic lamellas grids from GRP and 3D printed joints (Sect. 2.1), digitally design comparative asymptotic networks and simulate their kinetic behaviour with the ideal mechanical properties (Sect. 2.2). Both physical and digital models are actuated through five stages to determine the natural boundaries and behaviour of semi-compliant mechanisms. Mapping the total curvature gives insights into the structures' residual stresses and likely equilibrium states. In Sect. 3, this method is applied to 7 open and closed asymptotic networks of incremental complexity, increasing the number of singularities and cylinders: A regular grid (used later for the adaptive shading façade) (A), a hypar grid (B), a regular rotational grid (C), the rotational grid (a scaled model of the Kinetic Umbrella) (D), a Schwarz D grid with one singularity (E), a tricylinder grid with two singularities (F) and a quad-cylinder with three singularities (G). Finally, in Sect. 4, we introduce two prototypical architectural applications, an adaptive shading facade and the Kinetic Umbrella that show the possible scale and actuation for kinetic designs.

2 Comparative Modelling

This study is based on a hybrid physical and digital exploration (Fig. 3). This comparative modelling strategy allows input and feedback from both worlds at any time of the investigation.

2.1 Physical Modelling

Design and Assembly. The physical experimental models are designed and constructed from glass fibre reinforced plastic lamellas (1 × 10 mm) and joined laterally on two levels using 3D-printed sleeves. M3 steel bolts (20 mm) create the scissor joints, which allow rotation around the normal axis. The joints are located with a 3 mm offset to the theoretical intersection points due to the lateral connection. The positions are marked by hand and the 3D joints are threaded and glued onto the glass fibre lamella. The physical models are not replicas of a digital design but were used actively to find suitable networks, adjust their density and learn about the assembly process. Samples A and C were designed without any digital help based on pragmatic regular flat and cylindrical patterns.

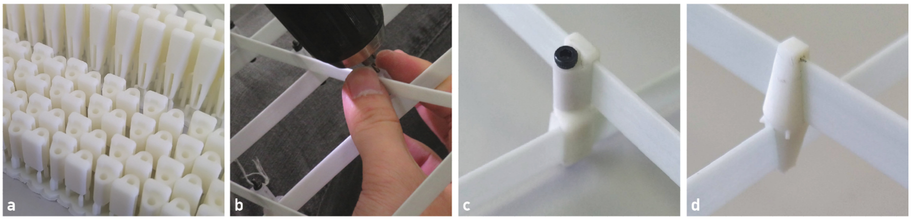


Fig. 4. The GRP lamellas were connected by hand (b) with 3d printed sleeves and M3 bolts (c). The sleeves are batch-printed (a) with a Form 3 Stereolithography (SLA) printer. A second, rigid sleeve (d) is used as internal restraint.

Actuation. The elastic grid is used to test various supports, constraints and displacements that control the kinetic behaviour. These physical tests are the basis for the digital simulations and development of actuation systems. Once a controlled kinetic behaviour is identified, the morphology is explored to its natural boundaries, which are marked by one of three occurrences (see also Fig. 7, Fig. 8, Fig. 9):

- **Planarity.** Sample A, B, C, and E are bounded by a planar state, which marks a point of symmetry in the kinetic behaviour. From here, the inverse transformation can be expected.
- **Self-collision.** Sample C, D, F and G display a limit state, in which lamellas collide and hinder further movement.
- **Resistance.** All kinetic transformations (except for C) were naturally limited by the resistance of the material (torsion or bending). These boundaries were later adopted in the digital simulation.

The maximal movement is actuated by hand and systematically documented in plan and elevation.

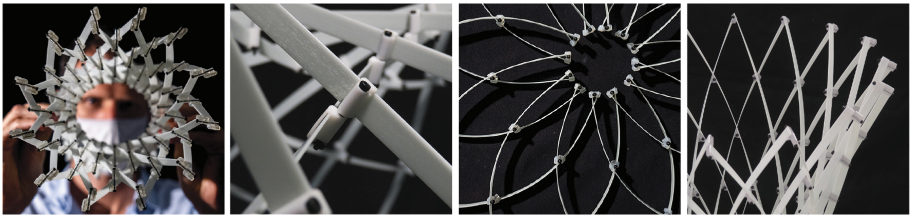


Fig. 5. The GRP models are tested extensively to determine suitable supports, constraints and displacements that control the kinetic behaviour.

2.2 Digital Design and Simulation

These physical experiments are simulated digitally based on the surface and network geometry, the stiffness parameters, support, restraints and actuation forces.

Design. In the first step, a reference surface is generated using either geometric operations (e.g. curve rotation for a rotational surface) or numerical optimization. We use isogeometric analysis (IGA) (Bauer et al. 2016; Oberbichler et al. 2019; Bauer 2020) to model the minimal NURBS surfaces of samples E, F and G. The IGA solver is embedded in Kiwi!3D plugin (Bauer and Längst 2019) for Grasshopper. Based on the reference surface, the asymptotic network is generated using the Bowerbird Pathfinder plugin for Grasshopper (Oberbichler 2019). The algorithm calculates the direction of asymptotic curves at any point on the surface and iteratively finds the path of zero normal curvature to draw two families of curves (Schling et al. 2017). Defining a homogenous network with appropriate density is straightforward for regular open surfaces (A, B) or rotational surfaces (C, D). In the case of complex surfaces with singularities (E, F, G) we use principal curvature lines, which bisect the asymptotic network, to define regular intersection points (Schling and Wan 2022).

This *initial design model* of surface and network can be used to create architectural visualizations, model the lamella geometry, define offsets, and find the intersection of curves. The length of curves and distance between intersections is the only digital data needed to mark straight strips of material and build a physical model.

Assembly. The initial design model is supplemented with ideal mechanical properties. For this purpose, we create an abstract discrete model of polygonal strips, which represent the intersections and lamellas. Each intersection is represented by a line i_i normal to the design surface which sits centered on the intersection point pt_i. Consecutive intersection-lines $(i_0, i_1, i_2, \ldots)$ are connected at top and bottom with lines t_i and b_i, creating polylines t and b above and below the asymptotic curves a. Together t, b and i form strips of quads, resembling the elastic lamellas (Fig. 6).

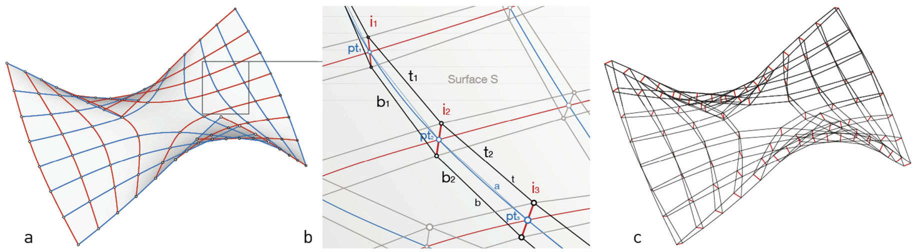


Fig. 6. Digital modelling. The initial, smooth design model (a) is rationalized (b) into a discrete polyline model (c) that allows embedding the mechanical properties.

We use the particle-spring solver Kangaroo 2 (Piker 2015) to embed simple optimization goals which resemble the ideal stiffness of profiles and rotation of joints. There are 6 *mechanical properties* (4 hard goals and 2 soft goals) that need to be embedded in the model.

Hard goals

- Resistance against **axial forces** (compression and tension): All lines i_i, t_i and b_i are restricted to their initial length.
- Resistance against **shear of the strong axis** of the lamella: The corners of the rectangle created by i_i, t_i, i_{i+1}, and b_i are restricted to 90 degrees.
- Resistance against **bending of the strong axis** of the lamella: The first two goals (length and angle) ensure that the two polylines t and b remain parallel and of the same length, and the quad-strip cannot curve up or down.
- **Scissor joints** with rotation limited to the normal axis: The points in which lines touch are treated as hinged connections by Kangaroo2. The intersection lines act as coupling of intersecting quad-strips and naturally limit the rotation to the normal intersection axis.

Soft goals

- Resistance against **torsion** of the lamella: The angle between consecutive intersection lines (i_i, i_{i+1}) is drawn towards 0. This motivates the intersection lines to become parallel and resist torsion of the polygon strip.
- Resistance against **bending of the weak axis** of the lamella: The angle between consecutive lines (t_i, t_{i+1}) and (b_i, b_{i+1}) is drawn towards 0. This motivates the quad-strips to become straight, i.e. minimize their geodesic curvature.

Constraints and Actuation. Once all mechanical properties are embedded, the Kangaroo solver is started, and the asymptotic grid will start transforming in real-time to assume a new equilibrium shape that optimizes the soft properties of minimal torsion and minimal geodesic curvature.

Additional external and internal constraints may be applied to control the kinetic movement. We divide these constraints into three groups:

- **Supports (external):** For all samples A-G the model was held in a symmetry plane to avoid global rotations or translations.
- **Restraints (internal):** Based on extensive physical testing additional restraints were added, which ensure a controlled movement and avoid distortion of the grid. E.g. for samples A and B, the rotation of specific intersection points was blocked to prevent in-plane shearing of the quadrilateral network.
- **Displacements:** To fully explore the complete kinetic movement of each sample, displacements are used to pull or push the structure beyond its equilibrium shape. These loads mimic either the external hand gestures or internal actuation systems used during physical experimentation.

This method is flexible and allows to carry out and adjust *kinetic simulation* on the fly to closely match the physical behaviour of networks.

Analysis. The kinetic behaviour is recorded in five stages. We can derive valuable geometric information from this sequence: Measuring the geodesic curvature k_g and geodesic torsion τ_g give insight on where the highest strain occurs and how they are distributed over the grid. The internal strain energy of the grid structure $\Pi_{i(t)}$ can be approximated using the total sum of curvature-squared (CS) and the beams stiffness parameters:

$$\Pi_{i(t)} = \frac{GI_T}{2} \int_c \tau_{g(t)}^2 ds + \frac{EI_y}{2} \int_c \kappa_{g(t)}^2 ds \tag{1}$$

The progression of the total CS sum throughout the transformation offers to predict the equilibrium state[1] and design the kinetic behaviour by adjusting the profile stiffness parameters. To measure these curvature values, the discrete model is transferred back

[1] This is valid, if the deformability of the grid is highly constrained, and thereby stiffness parameters do not affect the structure's transformation path.

to a smooth NURBS geometry, by interpolating the intersection points pt_i to recreate curves and surfaces with sufficient accuracy. We use the Plugin Bowerbird to analyse each "curve on surface", measure k_g and τ_g, create their total CS sums and map the results over five stages. The *CS-graphs* (Fig. 7, Fig. 8, Fig. 9) show the interpolated curve of the five CS results for geodesic torsion (in red) and geodesic curvature (in blue). Marked with a green line is the natural equilibrium state observed in the physical models. This state can theoretically be moved to any state within the green area, by adjusting the proportion of torsional to bending stiffness in the profile.

3 Morphology

We designed 7 open and closed asymptotic networks, to systematically investigate their kinetic behaviour (Fig. 7, Fig. 8 and Fig. 9). The data reveals fundamental relationships of the topology of networks with the kinetic boundaries, necessary constraints, equilibrium states and actuation methods.

Kinetic Boundaries. All open networks (A, B, and E) exhibit a boundary in the form of planarity. The planar state acts as symmetry plane from which an inverse kinetic behaviour is possible. The CS-graph can be mirrored at this state. Rotational grids (C and D) may also display a state of planarity where torsion is zero (C), and geodesic curvature becomes maximal. This creates a snap through effect (indicated with a green circle in the CS-graph). Any cylindrical network (C, D, F, G) will further exhibit a self-collision when the cylinders are closed and the grid approaches the cylinder axis. Other self-collisions were preceded by the resistance of the material.

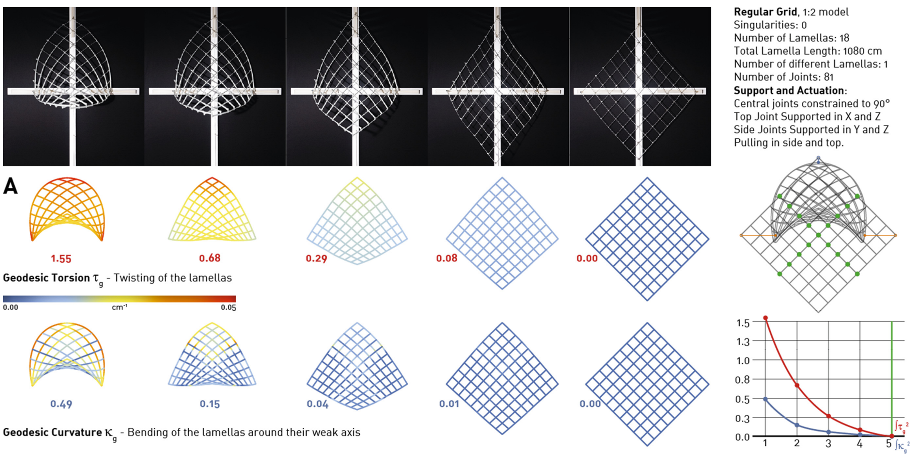


Fig. 7. Kinetic analysis for regular grid A. The grid is constrained by rigid joints (green dots). The outer corners are pulled inwards (orange lines) to actuate the double-curved shape. The planar state is the natural equilibrium (green line in CS-graph) where both geodesic torsion and curvature are zero. The graph can be mirrored here.

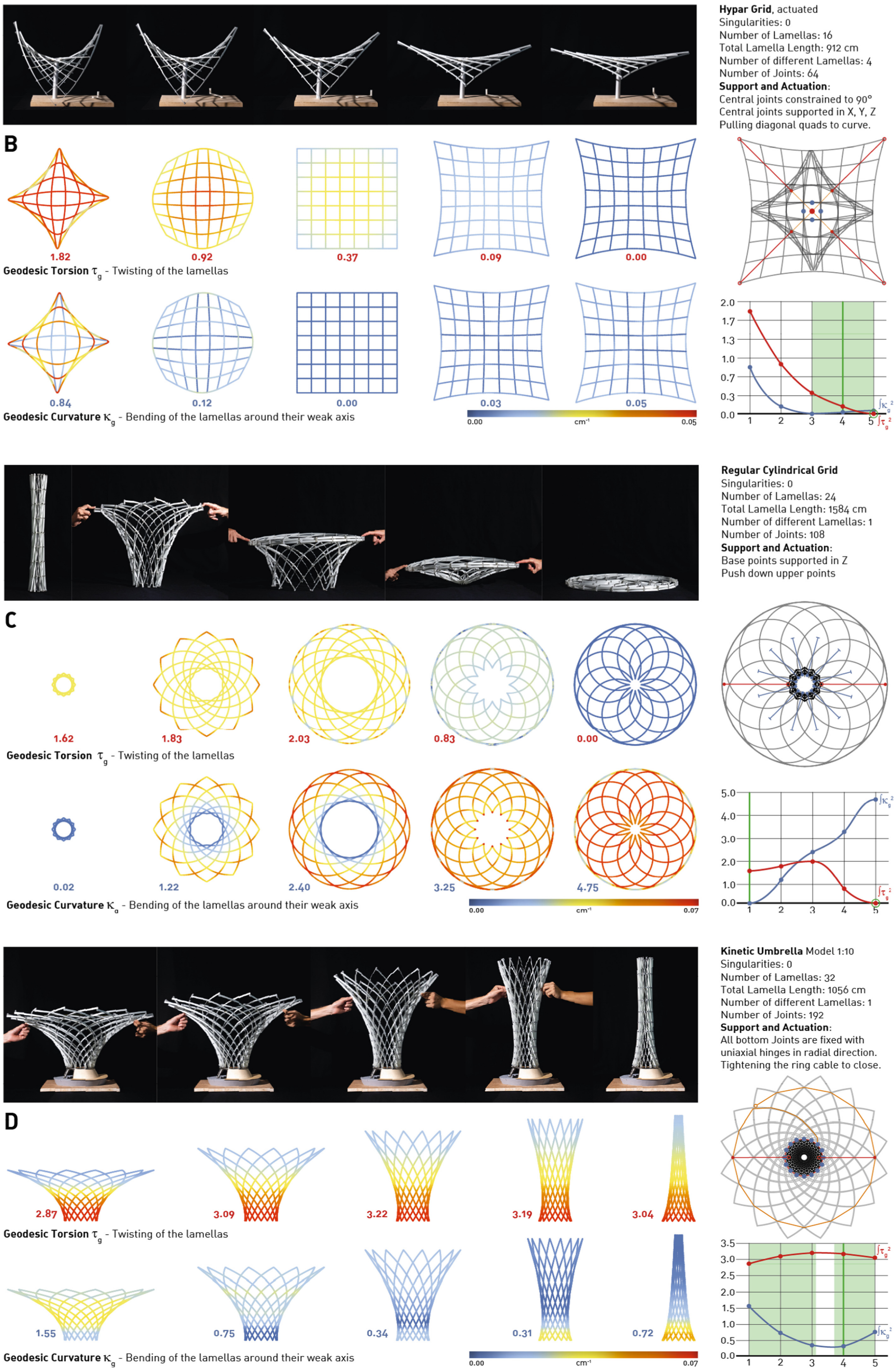


Fig. 8. Kinetic analysis showing the physical and digital models, constraints and CS-graph.

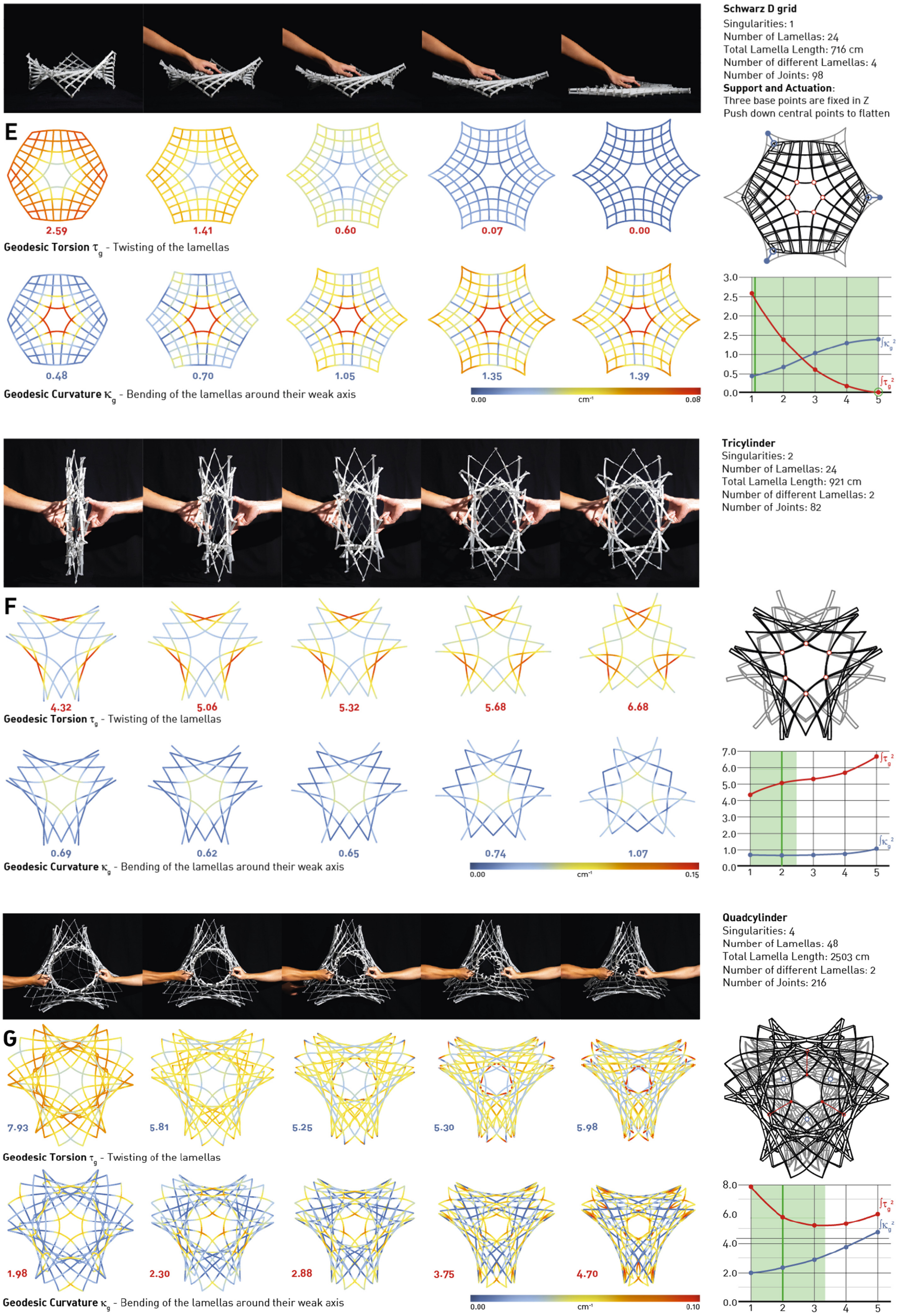


Fig. 9. Kinetic analysis showing the physical and digital models, constraints and CS-graph.

Constraints and Actuation. Open grids without singularities are initially unstable and require external and internal constraints to avoid a kinematic scissor/shear movement. In the case of the regular grid, the controlled kinetic transformation was achieved by adding rigid joints (see Fig. 4) along the two central lamellas (marked in green). In the Hypar model (B), the central square was fixed (Fig. 5, B) to prevent shearing. Cylindrical grids are less susceptible to shearing and tend to keep a rotational symmetry (C, D). They naturally shorten and increase their radius when pulled apart (C). The transformation can be controlled by adding pinned supports at one end (D). Singularities (E, F, G) act as restraints within a grid (like rigid joints) and create a controlled symmetric transformation. All three samples (E, F, G) were actuated by hand (marked in red) without further restraint, other than the natural supports (the table they were standing on, marked in blue). With increasing singularities, their transformation becomes more restricted. All samples presented in this paper, are design symmetrically, which contributes to their controlled movement.

Equilibrium. The natural equilibrium state (in which the energy is minimal) is dependent on the specific stiffness parameters of lamellas. For the regular open grid (A) planarity is the state of this equilibrium. For irregular networks (B), this equilibrium is offset from the planar state depending on the impact of geodesic curvature. The minimum-energy state of rotational networks tends to be near the closed position, where geodesic curvature is minimal. Singularities inevitably embed geodesic curvature in the grid and introduce reciprocity of bending and torsion. This increases the spectrum of possible minimum-energy states, depending on torsional and bending stiffness.

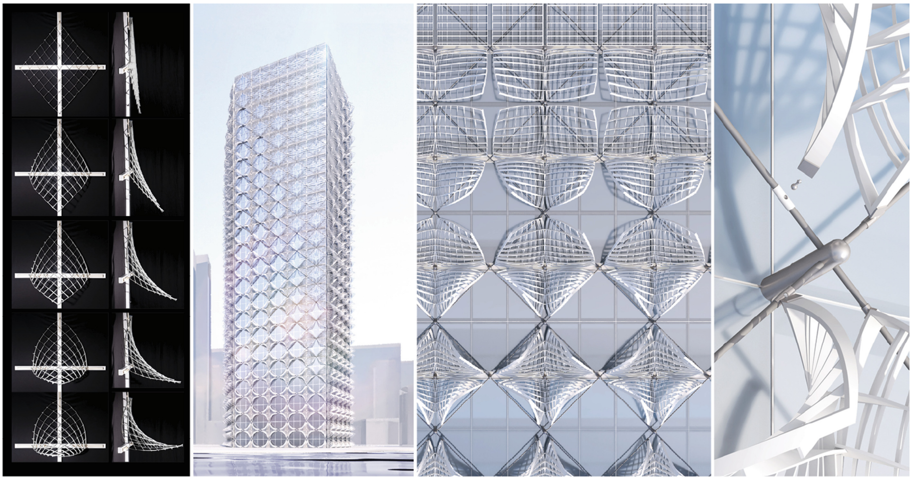


Fig. 10. Adaptive shading. The regular grid, system A, was developed for an integrated façade system that allows dynamic shading adjustment. The actuation is guided by diagonal rails.

4 Architectural Applications

The experiments are directly applicable to architectural practice. System A was further developed into an **adaptive shading system** (Fig. 10) to be integrated into existing building facades. The GRP lamellas are actuated by shifting the two diagonal corners inward transforming the regular grid into a doubly-curved canopy. The mechanism can be locked in any position to adjust to the sun angle.

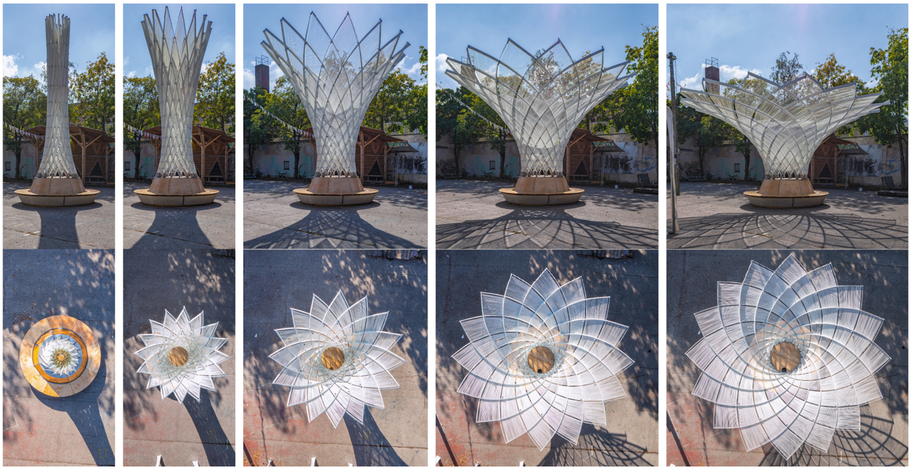


Fig. 11. The Kinetic Umbrella was completed in August 2021. The 6 m tall structure is actuated via a winch at the base and transforms from a cylinder into a blossom-like canopy of 8 m diameter.

The **Kinetic Umbrella** (Schikore et al. 2020) was completed in August 2021 at the at the "Kreativquartier" in Munich. The rotational asymptotic grid is based on system D. It was covered with an adaptive system of ribbons to create shading. The structure performs a reversible semi-compliant transformation from a slender cylinder of 6 m height (closed) to a funnel shape of 8 m diameter (open). Figure 11 shows the structure and shadow effect in five stages of transformation. In contrast to the scaled model D, this large-scale application naturally tends to open due to gravity, and can thus be actuated reversely by pulling the a ring cable and closing the cylindrical grid. The kinetic structure (Fig. 12) consists of two layers of 16 8 $\times$ 80 mm GRP slats, connected with aluminium joints and steel bolts (a). A textile cover of 10 mm ribbons provides shadow and adapts to the grid structure's transformation (a, b, c). Three circular fixed cables (yellow) are locking the transformation in the open state (b, d). The transformation is actuated by an additional, circular cable (red), leading down to a winch (f). The lamellas are connected via uniaxial hinges (e) to an octagon steel base that sits on concrete foundation bodies (d, f). Timber panels, covering the concrete and steel base, accommodate circular seating (f).

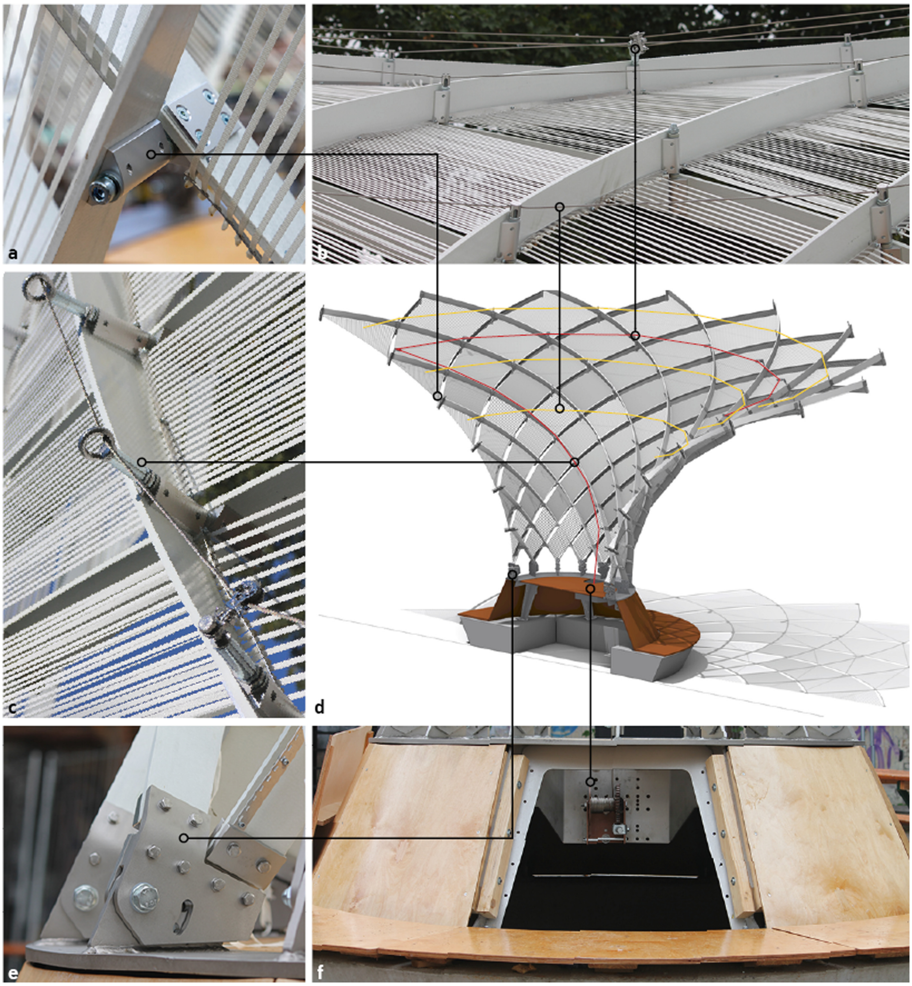
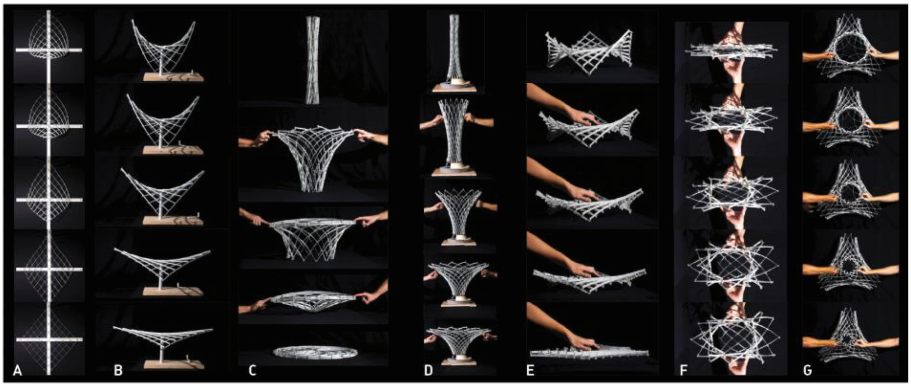


Fig. 12. Components and details of the Kinetic Umbrella. a: The lamellas are joined with aluminium profiles on two levels. b/d: Three ring cables (yellow) secure the opened structure. c/d: A fourth ring cable (red) actuates the closing movement. e: Uniaxial hinges constraint the base but allow a controlled movement. f: The red cable is pulled by a winch that is attached to the concrete base. Timber panels provide seating.

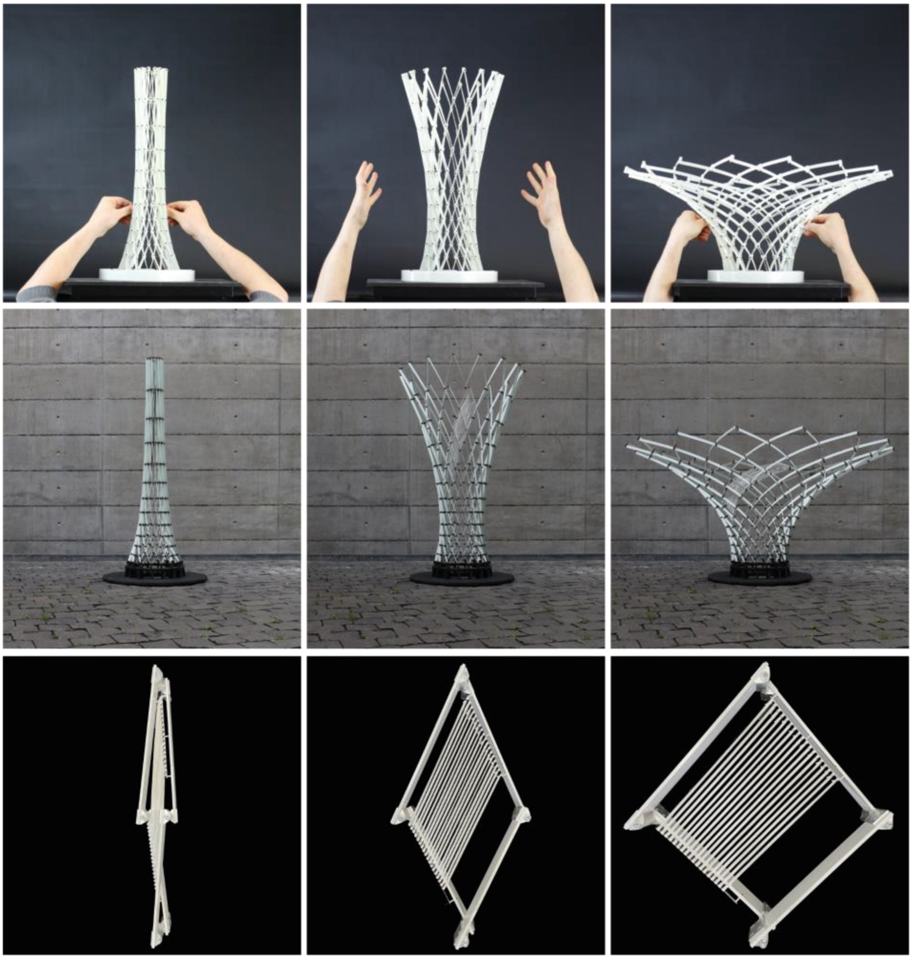
5 Conclusion

The hybrid approach of physical and digital experiments offers a broad investigation of complex kinetic systems while ensuring a realistic evaluation and application of results. The digital method proposed is fast and flexible, and accurately simulates the kinetic morphology of complex grids. The analysis is purely geometrical, as no actual material properties or profile dimensions are given. Nonetheless, measuring the geodesic curvature and geodesic torsion allows the prediction of residual bending and torsional stresses. Mapping the total curvature-squared gives insights into the likely minimal-energy states of the grid. This method is applied to seven asymptotic networks, and systematically investigates the impact of topology, singularities and support conditions on the kinetic behaviour. The data reveals strategies to constrain the transformation of asymptotic grids. Open grids without singularities are most flexible and require additional constraints. Cylindrical grids offer a more directed movement and can be controlled through pinned supports. Singularities act as internal constraints and create symmetric kinetic transformations. However, with an increasing number of singularities and cylinders, the movement becomes more restricted.

Two prototypical kinetic designs, an adaptive shading facade and a kinetic umbrella structure show possible architectural applications, including actuation system and detailed constructive solutions. The use of slender lamellas offers elastic transformation around the weak profile-axis while maintaining high structural stiffness of the strong axis. For large-scale application, the effect of gravity influences the minimum-energy state, favouring geometries with less elevated mass.

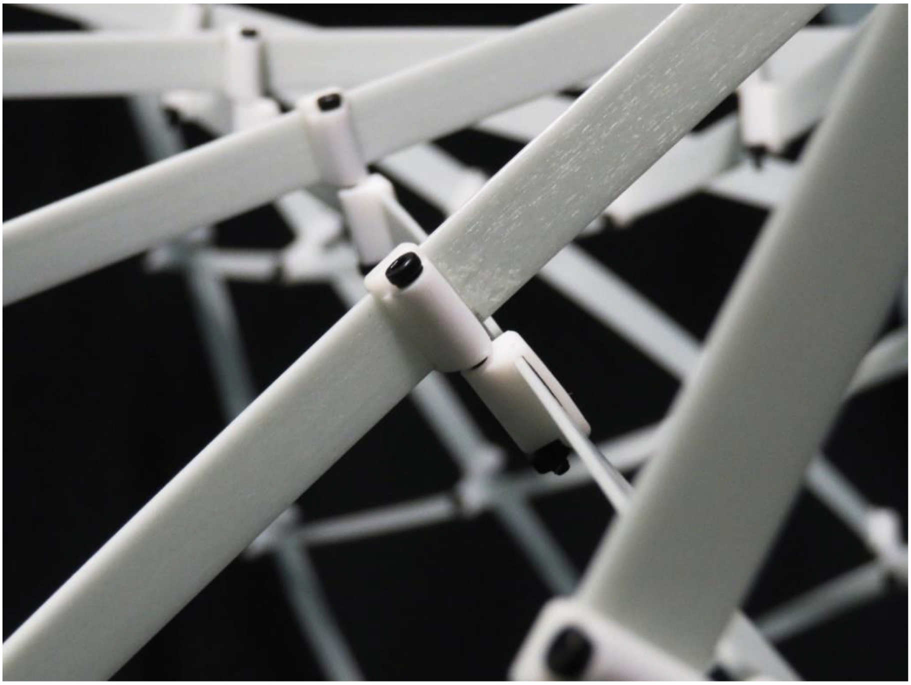


All actuated models in five phases.

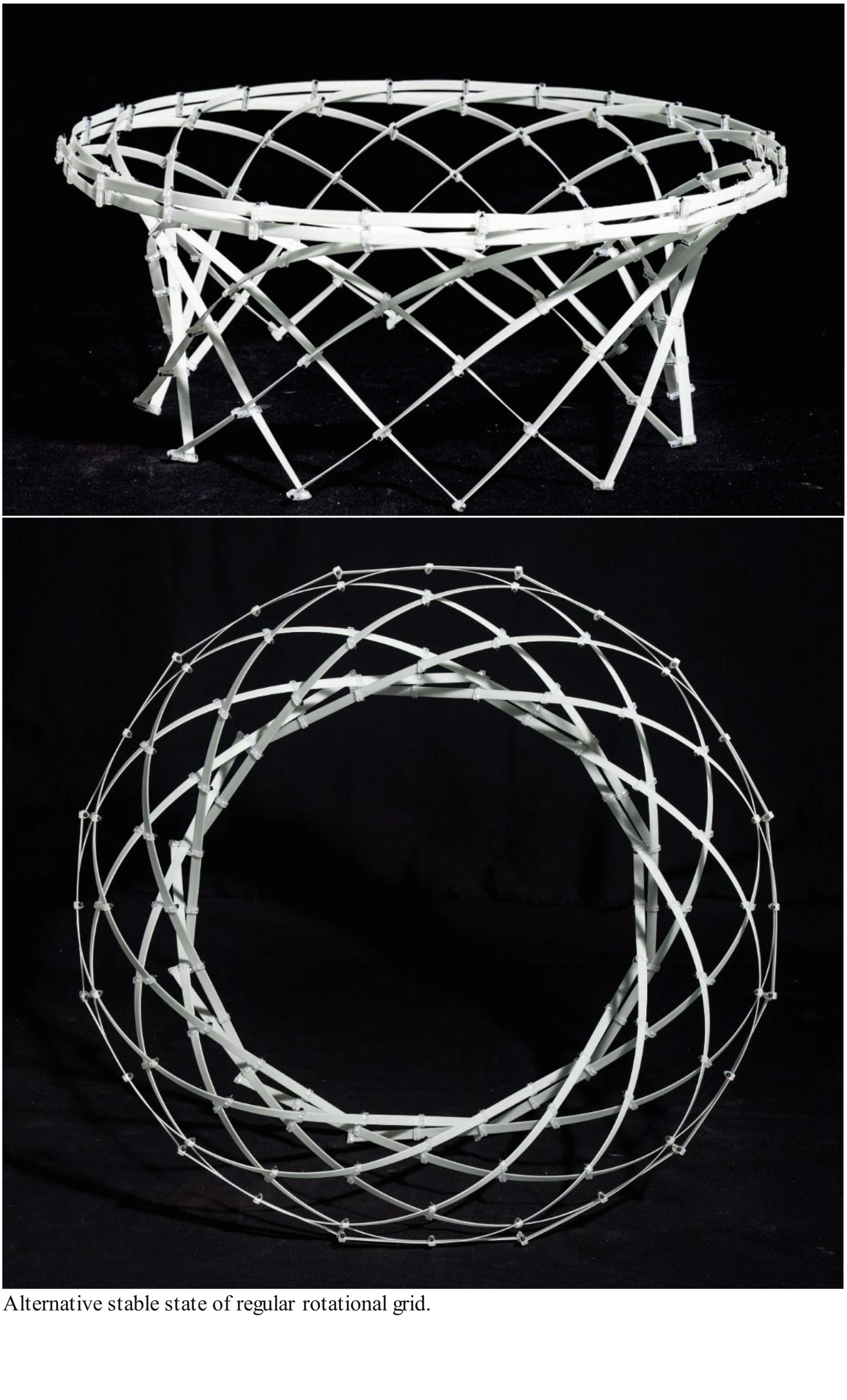


Physical models of the Kinetic Umbrella. Top, 1:10, Middle 1:3, Bottom 1:1

Close-up view of umbrella model



Close up of grid and joints



Alternative stable state of regular rotational grid.

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